Multi-Parameter Functions in Chaotic Dynamical Systems

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Table of Contents

Definition 1.8

Let $f : I \to J$ and consider $x \in I$. The point x is a *fixed point* for f if $f(x) = x$.

Definition 1.9

Let $f : I \to J$ and consider $x \in I$. The point x is a *periodic point* of period n if $f^n(x) = x$. We denote the *set of periodic points* of period n by $Per_n(f)$.

Definition 1.10

Definition 1.13: If f is a homeomorphism, we may define the *full orbit* of x, $O(x)$, as the set of points $f^n(x)$ for $n \in \mathbb{Z}$.

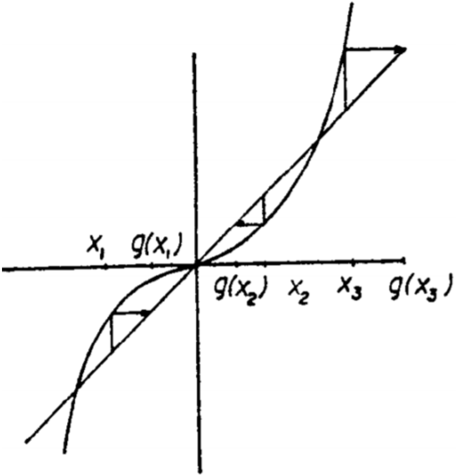

Figure 1.1:

Iteration map of $g(x) = x^3$.

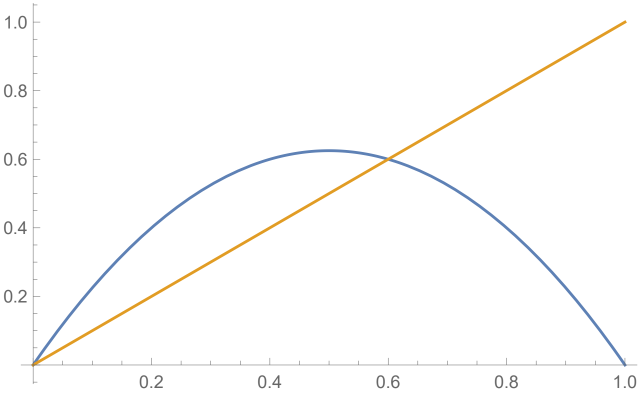

Figure 1.3:

Graph of $F_\mu(x) = \mu x(1-x)$ when $\mu = 2.5$.

We will first define Λ:

1. $A_1 = \{x \in I | F(x) \in \mathcal{A}_0\}$.
2. $A_n = \{x \in I | F^n(x) \in A_0\}$ $=\{x \in I | F^i(x) \in I$ for $I \leq n$ but $F^{n+1}(x) \notin I\}$
3. $\Lambda = I - \left(\bigcup_{n=0}^{\infty} A_n\right)$
    1. Λ consists of only the points which never escape from I
    2. $I - \left(\bigcup_{n=0}^{n-1} A_n\right)$.

Theorem 1.22

If $\mu > 2 + \sqrt{5}$, then Λ is a Cantor set.

Definition 1.37

Let V be a set. $f : V \to V$ is said to be *chaotic* on V if

1. f has sensitive dependence on initial conditions.
2. f is topologically transitive.
3. periodic points are dense in V.

Example 1.39

The quadratic maps $F_\mu(x) = \mu x(1-x)$ are chaotic on Λ when $\mu > 2 + \sqrt{5}$.

New Function

$$f_{a,b}(x) = \begin{cases} 4ax & 0 \leq x < 0.25 \\ 4x(b-a) + 2a - b & 0.25 \leq x < 0.5 \\ 4x(a-b) + 3b - 2a & 0.5 \leq x < 0.75 \\ -4ax + 4a & 0.75 \leq x < 1 \end{cases} \tag{1}$$

We chose to focus on the parameters where and $0 \leq a < 1$ and $-1 < b < 0$.

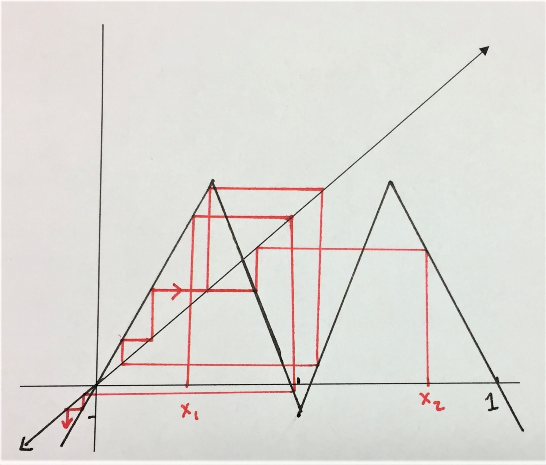

Figure 2.8:

Iteration map of $f_{0.5,-0.1}(x)$.

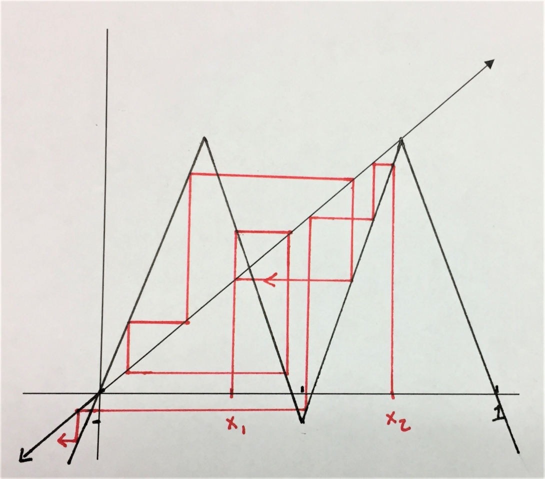

Figure 2.9:

Iteration map of $f_{0.75,-0.1}(x)$.

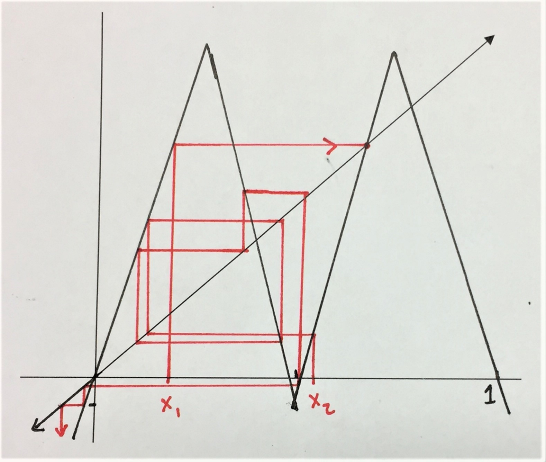

Figure 2.10:

Iteration map of $f_{0.9,-0.1}(x)$.

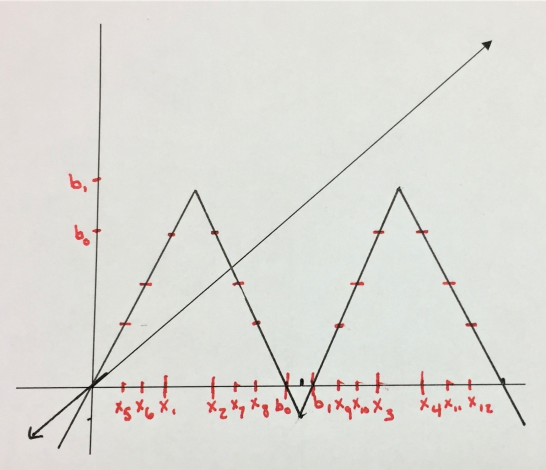

Figure 2.11:

Interval graph of $f_{a,-0.1}(x)$ with $0.4549 \approx b_0 < a < b_1 \approx 0.5391$.

Results

Theorem 2.1

For $b = -0.1$ and $0.4549 \approx b_0 < a < b_1 \approx 0.5391$ the invariant set of $f_{a,b}(x)$ is a closed and totally disconnected set.

1. We conjecture that Λ is a perfect subset of I. If this is true Λ would be a Cantor set.
2. Finally we conjecture that our function $f_{a,-0.1}(x)$ maps are chaotic on Λ when $0.4549 \approx b_0 < a < b_1 \approx 0.5391$.

Conclusion

1. Our final concluding discoveries are about:

$$f_{a,b}(x) = \begin{cases} 4ax & 0 \leq x < 0.25 \\ 4x(b-a) + 2a - b & 0.25 \leq x < 0.5 \\ 4x(a-b) + 3b - 2a & 0.5 \leq x < 0.75 \\ -4ax + 4a & 0.75 \leq x < 1 \end{cases} \tag{3}$$

2. If $0.4549 \approx b_0 < a < b_1 \approx 0.5391$ and $b = -0.1$ then the points remaining in I form a closed and totally disconnected set.
3. We conjecture that this invariant set is also a perfect subset of I, making the set itself a Cantor set.
4. We conjecture that under these parameters f maps are chaotic on their invariant set.

Thank you!

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